BY USING METHODS THAT REFLECT THE PROBABILITY OF PREDICTED COMBINATIONS OF MARKETING PERIOD AND PRICE VOLATILITY VARIABLES, PRACTITIONERS CAN CRAFT DLOMS THAT ARE SPECIFIC TO VALUATION SUBJECTS AT PARTICULAR TIMES.

CALCULATING PROBABILITY-BASED

1+596+00 91+00 149+00 1+594+961+00 16+456+00 64+00 5+516+156+00 155+00

8+195-00 1+649+00

> 6+00 5+00 19+00

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DISCOUNTS FOR LACK OF MARKETABILITY



represents the ability to sell an investment quickly when the investor decides to sell. Conversely, then, lack of liquidity represents the cost of failing to realize gains or failing to avoid losses on an investment during the period in which the investor is offering it for sale. A discount for lack of marketability (DLOM) should reflect the volatility of the value of the investment during the period of time that it is being marketed.

Transactional Marketing Periods

The business valuation concept of marketability deals with the liquidity of the ownership interest.¹ How quickly and certainly an owner can convert an investment to cash represent two very different variables. The quickly variable represents the period of time it will take the seller to liquidate an investment. This period of time can vary greatly depending on the standard of value in play. For example, liquidation sales can occur quickly and reflect lower prices. Orderly sales usually take longer to explore the marketplace of reasonable buyers, and generally reflect higher prices. In every instance, however, the quickly variable commences with a decision by the seller to initiate the sales process.

The marketing period of a privately held business is seldom less than a few months, and can be much longer, as the following events occur:

- Drafting selling documents.
- Developing a marketing strategy.
- Implementing the marketing.
- · Screening buyers.
- · Conducting site visits.
- Assisting buyers in their analysis of the company and the interest being sold.
- Drafting letters of intent.
- Negotiating with the serious buyers.
- · Assisting buyers with due diligence.

the seller will realize the estimated sale

• Actually closing the deal.

price (value) of the investment. Therefore, the certainty variable represents the price volatility of the investment during the period that it is being offered for sale. If market prices for similar investments fall dramatically while the marketplace is being explored, then the seller will have lost the opportunity to lock in the higher price that existed at the time the sell decision was made. Conversely, if the sale price is fixed for some reason (e.g., a listing agreement) and market prices for similar investments rise dramatically during the marketing period, the seller will have lost the opportunity to realize the increased value.

· Participating in arranging financing.

tainty, represents the probability that

Certainty. The second variable, cer-

The quickly and certainty variables work together when determining the value of an investment. Relative to immediately marketable investments, the value of illiquid investments (regardless of the level of value) must be discounted to reflect the uncertainty of the time and price of sale. This uncertainty is reflected in business valuations by the DLOM.

Price Risk

Logically, the economic costs of time and price uncertainty can be reduced to the price risk faced by an investor, during the particular period that an illiquid investment is being offered for sale. Investments that are immediately marketable can be sold at the current price to avoid the risk of future volatility. The illiquidity experienced by the seller of a non-public business interest during the marketing period, however, represents an economic cost. This cost is reflective of the risk associated

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with the inability to realize gains and avoid losses during the period of illiquidity.² The longer that time period, the more the value of the business is exposed to adverse events in the marketplace and adverse changes in the operations of the business, and the greater the DLOM that is required to equate the investment to an immediately liquid counterpart.

Longstaff Formula. The economic cost associated with a period of illiquidity can be estimated using the look-



back formula developed by Dr. Francis Longstaff in 1995,3 which relies on estimates of price volatility (i.e., the certainty variable) and marketing time (i.e., the quickly variable). Longstaff's approach, and the criticisms that have been directed at his methods, are discussed further below.

Volatility

Price volatility is easily determined if the appraiser can identify at least one appropriate publicly traded company

to use as a benchmark.⁴ This is obviously a matter of professional judgment. At the present author's firm, the same companies are used for price volatility determination as are used to apply the publicly traded guideline valuation method. The annualized average stock price volatility and standard deviation are calculated for each of the guideline companies for an historic period that is considered to be predictive of the time it will take to market the interest being valued.⁵ The calculated



 Pratt and Niculita, Valuing a Business: The Analysis and Appraisal of Closely Held Companies, 5th Ed. (McGravv-Hill, 2007), page 417.

2 Id.

- 3 Longstaff, "How Much Can Marketability Affect Security Value?," 50 J. Finance 1767 (December 1995).
- 4 The use of guideline companies to estimate the subject company's stock price volatility is consistent with the requirements of Statement of Financial Accounting Standards (SFAS) No. 123(R) at paragraph 23 and A22.
- 5 Subject to possible adjustment described in SFAS No. 123(R), using the historical volatility of stock over the most recent time period corresponding in length to the expected period of restriction is consistent with the requirements of the pronouncement. See paragraph A21.

EXHIBIT 1 Average Marketing Periods







EXHIBIT 3 SIC Code Divisions

SIC Code	SIC Group	Number of Sale Transactions	Average Selling Time in Days
01-09	Agriculture, forestry, and fishing	269	182
10-14	Mining	7	187
15-17	Construction	379	239
20-39	Manufacturing	927	216
40-49	Transportation, communications, electric, gas, and sanitary services	248	199
50-51	Wholesale trade	510	219
52-59	Retail trade	2,949	197
60-67	Finance, insurance, and real estate	152	193
70-89	Services	2,741	191
91-99	Public administration	2	<u>246</u>
	All industries	8,184	200

means and standard deviations volatilities are then averaged using a simple average or harmonic average as called for by the valuation purpose.⁶

Simple averages are generally favored when applying guideline factors in business valuation because the goal is to determine the fair market value of a particular investment. Harmonic averages may be useful, how-

Marketing Periods Study

To evaluate the period of time that it takes to sell privately held businesses, a database of 8,184 private company sale transactions was obtained from BV Resources.⁷ The database reported:

- An associated Standard Industrial Classification (SIC) code.
- Sale initiation date.

Exhibit 1 shows that the population of sale transactions follows a logarithmic distribution. The peak of the graph is 1,032 sale transactions that occurred from 30 to 59 days to sell, which is 12.6% of the database.⁸ The database analysis indicates that one standard deviation to the right of the mean encompasses marketing periods of up to 395 days, which is 88% of the database population.



ever, if the goal is to create a portfolio of investments that mirrors a particular market. Regardless of the averaging convention selected by the appraiser, basing price volatility estimates on guideline company stock price fluctuations eliminates the "upper bound" objections that some critics have of the Longstaff formula (discussed further below), by yielding a discount reflective of *average* price volatility instead of *peak* price volatility.

As with guideline company selection, the methodology for predicting future price volatility requires professional judgment. Appraisers may reasonably employ other ways of predicting price volatility than described above.

- Sale closing date.
- Market value of invested capital (MVIC).
- Asking price.

The average time that elapsed from the initial offering date to the closing date of these transactions is 200 days. The standard deviation of the reported time periods is 97.7%, or 195 days. The graph in Exhibit 1 shows the distribution of the amount of time it took to consummate the sale transactions in the database. Because the marketing time period cannot be less than zero days, the distribution of the database obviously skews to the right. The data is split into 30-day increments for presentation and analytical purposes.

- 6 On occasions, the volatilities of the guideline companies are averaged using a weighted average that reflects the companies' relative participation in the industry of the subject company.
- 7 Pratt's Stats is the BV Resources database where the transactions were obtained. The accuracy with which transactions are reported in the database was not investigated.
- 8 When the sales are presented on single-day time periods, spikes in the frequency of sales transactions occur about 30 days apart. This could be the result of faulty information supplied by brokers, or a tendency of sales to occur at the end of listing agreements. The use of 30-day periods eliminated the distortion of the spikes.
- 9 A log-normal distribution is positively skewed, with most values near the lower limit, and is based on natural logarithms.
- 10 The 89.2546% "certainty" shown in Exhibit 2 is not a probability certainty. Instead it is an absolute measure of the percentage of the population represented by one standard deviation to the right of the mean. See the Oracle Crystal Ball User's Guide at p.100.

This analysis was then compared to a distribution created using the population's mean and standard deviation and Oracle's Crystal Ball software. Exhibit 2 shows the Crystal Ball output using a log-normal distribution.⁹

Exhibit 2 shows that the peak frequency of sale events is 5.9%, which occurs from the range of approximately 64.2 to 76.6 days. But Exhibit 2 is based on 12-day, not 30-day, intervals. Adjusted ratably to a 12-day interval, the peak probability of Exhibit 1 is 5.0%. And as with the actual database, the Crystal Ball analysis indicates that one standard deviation to the right of the mean encompasses marketing periods of up to 396 days, representing 89% of the database population.¹⁰ Therefore, the database population follows the log-normal distribution of Crystal Ball, which is used for the remainder of this analysis.

Marketing Periods Based on Industry

Digging deeper for an industry analysis, the sale transactions were separated into the ten two-digit SIC code divisions corresponding to the broad industry groupings shown in Exhibits 3 and 4. The description, number of private sale transactions, and average days to sell is listed for each industry group. The standard deviations of these industries range from 143 days to 257 days. Exhibit 4 graphically depicts the variation in the averages from Exhibit 3.

The 57-day spread between the 239day average selling period of construction businesses and the 182-day average selling period of agriculture, forestry, and fishing businesses demonstrates that industry makes a material difference in how long it is likely to take to close the sale of a business. Adding widely varying standard deviations of marketing periods to the various mean marketing periods of different industries highlights the very different marketing period risks faced by owners of businesses engaged in different industries.

Marketing Periods Based on Sale Year

The next factor explored is the effect on the marketing period of the calendar year in which the businesses were





E XHIBIT 5 Calendar Year Analysis					
lf Listed In	2007 Study Average Selling Time in Days	2008 Study Average Selling Time in Days	Number of Transactions		
1996	265	267	71		
1997	240	239	133		
1998	211	223	250		
1999	204	206	270		
2000	218	226	372		
2001	200	209	440		
2002	172	182	519		
2003	178	189	521		
2004	175	185	737		
2005	189	208	748		
2006	195	220	819		
2007	166	220	1,112		
2008		202	948		
Average	201	214			



listed for sale. The BV Resources database reports sale transactions commencing in 1991 and extending through 2010. The years 1991 through 1995 were not used in this analysis since there were very few listings from these years. Calendar years 2009 and 2010 were also not used because the closing dates of these listings were not yet known. These exclusions reduced the database population from 8,184 to 6,940.

Exhibit 5 shows the average marketing period and number of transactions by year for sales listed from 1996 through 2008.

Exhibit 6 shows the declining trend of average selling periods over time. The average number of days it took to sell the privately held businesses in the study decreased from 267 days in 1996 to 182 days in 2002, before increasing to 220 days in 2007 and falling to 202 days in 2008. The present study suggests that annual fluctuations in inflation, real GDP, nominal GDP, money supply, and demographics provide little explanation of the declining trend of private business marketing periods.

Recession. During the period analyzed, there was a recession from March to November in 2001. This possibly explains the longer selling times for those sales that were listed in 2000 and closed in 2001, but the explanation is seemingly contradicted by the decline in the average number of days to sell businesses listed in 2001. Despite the recession, the average business sold faster during 2001 than in 2000. A major recession also started in December 2007. This possibly explains the longer selling times for those sales that were listed in 2006 and 2007, which were both 5.8% longer than sales listed in 2005. One might expect these listings to take longer to close if they were initiated but not completed by the start of the recession.

Marketing Periods Based on Price

The transactions database also provided the MVIC and asking price of each transaction. These factors were used to separately analyze the database. The MVIC is the market value of invested capital comprised of all stock classes and interest-bearing debt.

The range of MVIC was \$1,000 to \$314 million. The mean and median MVIC of the population was \$783,067 and \$205,000, respectively. The sale transactions were split into 20 groups based on MVIC. The MVIC range of the group intervals becomes larger as MVIC increases. Each size group conthe MVIC groups yielded a fairly strong R-square of 73%. The regression formula shows that the average days to sell increases by 1.6% as MVIC progresses from group to group. The trend line predicts 171 days to complete a sale transaction when the MVIC is below \$40,000. When the MVIC is above \$2.35 million, the trend predicts at 231 days to sell, but the actual mar-



tains 409 sale transactions except for the largest group, which contains 413. Exhibit 7 shows the average days to sell for each MVIC group.

Generally, the average days to sell increases with the rise in MVIC. When the MVIC is under \$40,000, the average days to sell is 173 days. The length of marketing period gradually increases until the MVIC price is greater than \$2.35 million, when the average days to sell is 269 days.

Regression. Exponential regression of the average marketing periods of

keting time of this group is much higher as the graph in Exhibit 7 shows.

Some of the transactions did not report asking prices. Those sale transactions were removed from this analysis, which reduced the database population for the asking price analysis from 8,184 to 7,607. The mean of the 7,607 sale transactions is 196 days to sell. The range of asking prices of the resulting population of sale transactions was from \$3,456 to \$70,000,000. The mean and median asking prices of the population were \$608,018 and \$249,000, respectively. Each size group contains 380 sale transactions except the largest group, which contains 387. Exhibit 8 shows the average days to sell for each asking price group.

Fluctuations. The fluctuations in the asking price graph are generally similar to those of the MVIC graph in Exhibit 7. When the asking price is under \$55,000, the average days to sell

EXHIBIT 7 MVIC Price Analysis



EXHIBIT 8 Marketing Period Averages by Asking Price



e market-

EXHIBIT 9

If Listed In	Number of Sale Transactions	Average Days to Sell	
January	774	192	
February	682	204	
March	740	201	
April	697	190	
May	686	200	
June	697	195	
July	694	203	
August	678	214	
September	642	206	
October	689	207	
November	611	194	
December	594	193	
Average		200	

is 164 days. The length of the marketing period gradually increases until the average days to sell is 265 days when the asking price is greater than \$2 million.

Exponential regression of the average asking price of each group resulted in a strong 86% R-square. The regression formula shows that the average days to sell increases by 1.9%, as asking price progresses from group to group. The regression predicts that it takes 163 days to complete a sale transaction when the asking price is below \$55,000. When the asking price is above \$2 million, the regression predicts that it takes 232 days to close a sale. However, note that the 265-day average marketing period for businesses priced higher than \$2 million is significantly above the trend number.

As mentioned, the asking price regression yields a stronger R-square of 86%¹¹ while the MVIC regression yields a weaker 73%. The higher R-square valExhibit 10 depicts the variation in the calendar month averages from Exhibit 9. On average, sale transactions originally listed in August took the longest time to sell, with a mean of 214 days. March listings had the highest volatility of time to sell. Sale transactions originally listed in October also were lengthy, averaging 207 days to sell. The months with the with otherwise perfect market timing ability. But Dr. Longstaff also recognized that the value of marketability, and therefore the cost of illiquidity, is less for investors with less than perfect market timing ability. Consequently, Dr. Longstaff described his approach as the "upper bound" of DLOM calculations. Since 1995, criticisms of what is now known as the



ue associated with asking price may be due to reporting inaccuracies that were not investigated. But it may also reflect that asking price is determinative in drawing potential buyers to the sale opportunity. Assuming no database adjustments are warranted, the asking price is the better statistical predictor.

Role of Seasonality

This study also considered whether the time of year a sale transaction is initiated makes a difference in the length of marketing periods. To analyze this factor, the sale transactions were grouped based on the month the company was listed to sell. Exhibit 9 reports the mean number of days to sell that elapsed from the listing date based on a distribution of the sale transactions according to the calendar month the businesses were listed for sale. shortest marketing periods were April, January, December, and November averaging 190, 192, 193, and 194 days, respectively. Possible explanations for these phenomena are proximity to year-end numbers for November, December, and January listings, and proximity to completion of tax filings for April listings. Such proximity tends to offer buyers enhanced transparency through timelier financial reporting.

Rebutting Longstaff's Critics

In 1995, when Longstaff presented his idea that the formula for calculating the value of a look-back option with and without a liquidity restriction assumption could be used to estimate DLOM of a financial instrument, he described his approach as quantifying the cost of illiquidity for an investor Longstaff methodology have focused on three perceived defects:

- 1. No investor has perfect knowledge.
- 2. A DLOM based on an upper bound is excessive.
- 3. The look-back option formula "breaks down" under long marketing periods and high price volatilities.

Each of these criticisms is incorrect, however, for the reasons described below.

Perfect Knowledge Criticism. The "perfect knowledge" criticism is based on a defective definition of market timing in a valuation context. The context

¹¹ A linear regression resulted in an R-square value of 83%. The slope was 3.7, meaning for each increase from one asking price group to another, the average days to sell increases by 3.7 days.

¹² e.g., Vianello, "The Marketing Period of Private Sale Transactions: Updated for Sales through 2010," Business Valuation Update (November 2011).

considered by Dr. Longstaff was one of an investor looking back in time to observe precisely when an investment could have been sold at its maximum value. Dr. Longstaff implicitly assumed that the maximum price could have been reached at any point during the look-back period. But in a valuation context this reasonable assumption is not appropriate. Instead, the maximum price occurs on the valuation date and is the marketable value of the valuation subject. Appraisers determine this value in the ordinary course of their work.

Standing on the vantage point of the valuation date and applying lookback option pricing to calculate DLOM in a business valuation inherently assumes that the maximum price that the investor could have realized for the investment is the marketable equivalent price as of that date. The value of the investment beyond the valuation date is necessarily less. This is because the time value of money diminishes the present value of the marketable equivalent price over the course of the marketing period; the foreseeable favorable events affecting the valuation subject have been factored into the analysis; and investors are averse to the risks of price volatility. Thus, if the appraiser properly determined the marketable equivalent price as of the valuation date, then that price is the "maximum value" postulated by Dr. Longstaff.

Upper Bound Objection. Dr. Longstaff presented an investor with perfect timing abilities and described this framework, in which an upper bound on the value of marketability is derived, as one lacking the assumptions about informational asymmetries, investor preferences, and other variables that would be required for a general equilibrium model. He recognized that the cost of illiquidity is less for an investor with imperfect market timing than it is for an investor possessing perfect market timing. These considerations are the basis of the "upper bound" limitation of the Longstaff methodology.

It is irrefutable that the cost of illiquidity must be less for the average investor with imperfect market timing than it is for an investor possessing perfect market timing. But the "upper bound" criticism resulting from this sit-





uation is nonetheless defective in the valuation context because it is easily circumvented by using volatility estimates that represent average, not peak, volatility expectations. For example, the appraiser's volatility estimate may be based on some average or regression of historical price volatility derived from an index or from one or more publicly traded guideline companies. Using average volatility estimates in the look-back option formula necessarily results in a value that is less than the "upper bound" value. Indeed, a value calculated using average expected volatility necessarily suggests a result that is achievable by

the average imperfect investor. The resulting value appropriately determined in this manner falls short of a value based on perfect market timing, while providing an important informational asymmetry lacking in Dr. Longstaff's more simplified framework.

Enhanced estimates of DLOMs applicable to average investors can also be crafted by determining the average marketing period required to sell privately held businesses, and the standard deviation of distribution around the mean.¹² Using probability weighted marketing periods therefore provides a second important (*Continued on page 46*)

DLOM

(Continued from page 21) informational asymmetry lacking in Dr. Longstaff's framework.

Additional framework enhancements include determining the rate of incline or decline in future volatility, and weighting future volatility estimates according to the probability of sale associated with the time period in which the estimates are expected to occur. Accordingly, the "upper bound" criticism has no significance in a proper application of the Longstaff methodology.

The Formula "Breaks Down" Criticism. The IRS Discount for Lack of Marketability job aid13 makes the statement that volatilities in excess of 30% are not "realistic" for estimating DLOM using look-back option pricing models. In support of this contention, the publication provides a table reporting marketability discounts in excess of 100% resulting from using combinations of variables of at least 50% volatility with a five-year marketing period and 70% volatility with a two-year marketing period. When that occurs, Longstaff DLOM values should simply be capped at 100%. After all, the criticism is not that the formula incorrectly calculates DLOMs below the 100% limit; merely that DLOM cannot exceed 100%.

The graph in Exhibit 11 shows the Longstaff DLOM values, capped at 100%, that result from a 20% price volatility assumption and a broad range of marketing periods. The 20% price volatility assumption approximates the historical mean of the volatility index (VIX) from 1/2/1990, to 6/30/2011. Note that it takes about 6,970 days over 19 years—for the discount to reach 100% with a 20% price volatility assumption. Considering that the typical business sells in about 200 days, a criticism based on a 19-year marketing period is clearly unreasonable.¹⁴

Of course, as the expected price volatility increases, a shorter time is





required to reach 100%. Conversely, as the expected price volatility decreases, a longer time is required to reach 100%. The graph in Exhibit 12 shows the line demarking varying combinations of price volatility and marketing periods above which Longstaff DLOM values exceed 100%. Considering that the peak volatility of the VIX was about 80% (occurring on 11/20/2008) and that the average period of time in which a private business sells is about 200 days, it is unlikely that typical appraisers will define look-back option variables that result in Longstaff DLOM values that exceed 100%.

Using the Longstaff Model

Vianello Forensic Consulting, LLC recently launched a calculator¹⁵ to

provide practitioners with DLOM values that are based on outcome probabilities as discussed above. While detailed discussion of the calculator's many integrated features are beyond the scope of this article, the development of this program demonstrates that an informative and defensible probability-based DLOM can be obtained. The calculator uses Longstaff's look-back model in calculating DLOM. His formula is presented in Exhibit 13. The two variables required by the model are (1) marketing period and (2) price volatility.

Probability Estimates. The calculator gives users a probability distribution of the subject's marketing period. As discussed above, the marketing period of privately held businesses is influenced by industry, price, size, list-

¹³ Job Aid for IRS Valuation Professionals, Discount for Lack of Marketability, 9/25/2009.

¹⁴ The VIX peaked at 80.86% on 11/20/2008. With that assumption, the Longstaff formula requires a 450-day lock-up period to reach 100% DLOM.

¹⁵ The VFC DLOM Calculator, http://dlomcalcula tor.com.

ing month, and listing year. The estimator uses data from BizComps to calculate the mean and standard deviation marketing period based on the whole database as well as subsets of the population corresponding to the aforementioned influencers. Each subset has a number of sub-parameters, and each sub-parameter has an associated mean and standard deviation. If a single sub-parameter is selected, its mean and standard deviation are the basis for estimating the marketing period probabilities. If more than one sub-parameter is selected, the associated means and standard deviations are averaged, and the averages are the basis for estimating the marketing period probabilities.

Once a mean and standard deviation are determined a statistical modeling engine transforms them into a log-normal probability distribution depicting the probability that the asset to be valued will sell within a certain length of time. The calculator works similarly for estimating price volatility. The user can enter stock symbols for up to 20 guideline companies to use as benchmarks for volatility. The program will then calculate the mean and standard deviation of price volatility based on the companies provided by the user for look-back periods of 50 trading days, 100 trading days, 250 trading days, and 500 trading days. A statistical modeling engine then transforms the means and standard deviations into probability distributions depicting the probability that the asset to be valued will exhibit different volatility measures. An upper bound will be applied to the distribution at the point where the asset is 95% likely to have a volatility less than or equal to the value. The user will receive the mean, median, and mode of the probability distributions, probability graphs, and tables of the interval probabilities and cumulative probabilities that support the graphs for each of the different look-back periods.

Calculating DLOM. The calculator provides drop down lists to tailor marketing periods specific to the valuation subject based on relevant factors of industry, seasonality, year, employee count, asking price, and revenues. Or the practitioner can enter his or her own marketing period metrics. Likewise, the calculator aids the practitioner by automatically calculating price volatilities and standard deviations for guideline companies or indices. The practitioner can also enter his or her own price volatility metrics.

The DLOM for each marketing period and price volatility combination is calculated using the Longstaff lookback model. The DLOMs are next multiplied by the probability associated with each combination of marketing period and price volatility to produce a probability-weighted DLOM.

Conclusion

DLOMs seek to capture the risk associated with illiquidity. By applying the Longstaff look-back model using the VFC DLOM calculator (http://dlomcalculator.com), practitioners can craft DLOMs that are specific to the valuation subject and date, and reflect the probability of each predicted combination of marketing period and price volatility variables. ●

